

Asymptotically Schrödinger Space-Times

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We first review how asymptotically Schrödinger space-times arise in a natural way by performing a TsT transformation on asymptotically AdS space-times and some of its consequences. We then give a coordinate independent definition of a pure Schrödinger space-time in terms of an AdS metric and an AdS null Killing vector. Then, by analogy with the AdS case, we provide a local and coordinate independent definition of a Schrödinger boundary in terms of a defining function. We use this to construct the Fefferman–Graham expansions of locally Schrödinger space-times.

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1 Introduction

Asymptotically Schrödinger space-times form a natural starting point to try to extend holographic techniques to the realm of non-relativistic physics. The main reason for this is that Schrödinger space-times can be thought of as a deformation of AdS spaces and this deformation is mild enough such that certain properties of the AdS space directly carry over to the Schrödinger case. Clearly it is of great interest to fully take advantage of this fact in developing non-relativistic holographic techniques. One close connection between the Schrödinger space and AdS is that the former possesses as a symmetry group the non-relativistic analogue of the conformal group. This so-called Schrödinger group can be embedded into $SO(d+2, 2)$ and possesses an anisotropic scaling symmetry of the form $D_z : t \rightarrow \lambda^z t, x \rightarrow \lambda x$ where $z \neq 1$ is called the dynamical exponent and is such that the time coordinate scales differently than the spatial ones. There exists a non-relativistic counterpart of a CFT that is based on this group [1] and from a phenomenological point of view they could be of relevance to the description of cold atoms at unitarity [2, 3]. Next we will restrict ourselves to the special value $z = 2$ for which there is an extra special conformal symmetry and which is such that the metric admits a global timelike Killing vector field [4].

One of the reasons why it is challenging to develop holographic techniques for Schrödinger backgrounds comes from the fact that it is a non-distinguishing space-time [5, 6] that has an intrinsically Galilean like causal structure. The usual boundary proposals are not guaranteed to apply to such space-times. Indeed, one can show that the Penrose construction applied to the Schrödinger metric fails. Moreover, because on the causal ladder the space-time is only non-distinguishing boundary definitions such as the one given by Marolf and Ross [7] (see also [8]) are not guaranteed to work. It would be interesting to study if these methods can nevertheless be successfully applied to Schrödinger spaces. Horava and Melby-Thompson proposed an adaptation of the Penrose construction valid for anisotropic boundaries [9]. We will adopt a slightly different point of view (but leading to similar results) and show how a natural notion of a Schrödinger boundary emerges in terms of a defining function once the Schrödinger metric is expressed in terms of an AdS metric and a null Killing vector field. Our results on the Schrödinger boundary appear to have some overlap with [10].

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We mention that the last section was not part of the talk presented at the *XVII European Workshop on String Theory* in Padova, Italy, (5-9 September, 2011) by B.R. and is added to give a more complete picture.

2 Asymptotically Schrödinger spaces from TsT

Schrödinger solutions can arise in various gravitational theories but from the holographic point of view solutions embedded into string theory are the most relevant ones. It is well-known that $z = 2$ (asymptotically) Schrödinger solutions can be obtained as the result of a TsT (T-duality, shift, T-duality) transformation [11] performed on an (asymptotically) AdS space-time. Such a transformation can be thought of as a solution generating technique relating different solutions of type II supergravities. The TsT transformation will produce an asymptotically Schrödinger space-time (ASch) out of a solution of the form $\text{AAdS}_5 \times Y_5$ where the asymptotically AdS space (AAdS_5) is purely gravitational provided that the shift is performed along an isometry of the AAdS_5 that asymptotically becomes null and the T-dualities are done along an isometry of a Sasaki-Einstein internal space Y_5 . To actually perform the transformation requires adapted coordinates that make the two isometries involved, forming a 2-torus, manifest. Let us call ∂_φ the isometry on Y_5 , $\partial_{\tilde{\varphi}}$ its T-dual and ∂_V the asymptotically null isometry of the AAdS_5 space $g_{\mu\nu}$ along which the shift $V \rightarrow V + \gamma\tilde{\varphi}$ is performed. Then after a dimensional reduction one finds that out of the metric $g_{\mu\nu}$ the TsT transformation has produced the set of new fields [12]

$$\bar{g}_{\mu\nu} = e^{-2\Phi/3} (g_{\mu\nu} - e^{-2\Phi} A_\mu A_\nu), \quad (1)$$

$$A = \gamma e^{2\Phi} g_{V\mu} dx^\mu, \quad (2)$$

$$e^{-2\Phi} = 1 + \gamma^2 g_{VV}. \quad (3)$$

These fields together solve the equations of motion coming from the following action

$$I = \int_M d^5x \sqrt{-\bar{g}} \left(\bar{R} - \frac{4}{3} \partial_\mu \Phi \partial^\mu \Phi - V(\Phi) - \frac{1}{4} e^{-\frac{8}{3}\Phi} F_{\mu\nu} F^{\mu\nu} - 4 A_\mu A^\mu \right), \quad (4)$$

where the potential V is given by $V(\Phi) = 4e^{\frac{2}{3}\Phi} (e^{2\Phi} - 4)$. The solution (1)-(3) can always be uplifted to a solution of type IIB supergravity¹ [12]. Further, it can be shown that the isometries of $\bar{g}_{\mu\nu}$ are all those isometries of $g_{\mu\nu}$ that commute with the Killing vector ∂_V . Provided that this vector is (asymptotically) null it will be guaranteed that the metric $\bar{g}_{\mu\nu}$ (asymptotically) inherits the full set of Schrödinger isometries.

As a consequence of these observations a natural class of ASch space-times emerges. Namely, a class formed by the whole set of solutions obtained by applying a TsT transformation to those pure AAdS_5 space-times that admit a Killing vector ∂_V that becomes null asymptotically. It is then possible to identify a conformal class of Schrödinger boundaries that can be thought of as the TsT image of that subclass of AdS boundaries admitting a null Killing vector [13]. Moreover, it can be shown that the on-shell action (4) evaluated for the class of solutions (1) to (3) is TsT invariant meaning that it reduces to the same value the AdS action had before performing the transformation. This leads to important constraints for the construction of counterterms to the action (4). Indeed, whatever the complete set of counterterms for the full space of ASch_5 solutions is, upon substituting the solutions (1) to (3), the counterterms that contribute divergently must together equal the known counterterms of the usual AAdS_5 action. Furthermore, it can be seen that not only the on-shell action but actually also thermodynamic properties of TsT transformed AAdS_5 black holes such as entropy, temperature and chemical potentials are all TsT invariant [13].

All in all a lot can be learned by the study of asymptotically Schrödinger space-times produced by the TsT transformation. This is mainly to be attributed to the knowledge one has about AAdS spaces such as the structure of the boundary and Fefferman–Graham expansions which can be put to good use in

¹ The action (4) is a simplified version of a three scalar action that forms a consistent reduction of type IIB supergravity on a squashed 5-sphere [12].

deriving results for ASch spaces. Moreover, it is very appealing that these ASch solutions can be uplifted to solutions of type IIB supergravity. Nevertheless, TsT transformations have several drawbacks one would wish to cure. First of all, the transformation is intrinsically coordinate (and dimension) dependent as the shift isometry needs to be manifest. Another severe restriction comes from the fact that this isometry needs to be global rather than just an asymptotic one. This already suggests that the set of ASch spaces accessible via TsT is rather restricted and raises the question: Can we generalize the TsT construction? In order to answer this question we first need to formulate precisely what we mean by a Schrödinger boundary. This will be the subject of this proceedings paper in which we apply this to work out the Fefferman–Graham expansions for locally Schrödinger space-times. In an upcoming paper [14] we will then use this to construct Fefferman–Graham expansions of certain asymptotically Schrödinger space-times.

3 From AdS to Schrödinger

To cure some of the obstructions observed previously it is crucial to first reformulate the result of the TsT transformation in a coordinate independent manner. For this purpose we will now prove that a Schrödinger space-time $\bar{g}_{\mu\nu}$ can always be written in the form of a generalized Kerr–Schild metric [15]

$$\bar{g}_{\mu\nu} = g_{\mu\nu} - A_\mu A_\nu, \quad (5)$$

where $g_{\mu\nu}$ is the metric of an AdS space-time and A^μ any null Killing vector (NKV) field of $g_{\mu\nu}$ ². It then follows that (5) can be used as a coordinate independent definition for a pure Schrödinger space-time. What underlies this relation is that the Schrödinger algebra consists of all AdS isometries that commute with a NKV whereas at the same time under the shift only those isometries that commute with A^μ are preserved.

Proof. Let K^μ be an isometry of $g_{\mu\nu}$. It then follows from (5) that

$$\mathcal{L}_K \bar{g}_{\mu\nu} = -A_\mu \mathcal{L}_K A_\nu - A_\nu \mathcal{L}_K A_\mu, \quad (6)$$

where \mathcal{L}_K denotes the Lie derivative along K^μ . Note that if $[K, A] = 0$ then K^μ is also an isometry of $\bar{g}_{\mu\nu}$. Next we show that the Schrödinger algebra is formed by the commutator of any choice of AdS NKV. Namely, $\text{sch}(d+3) = \{X \in \text{so}(2, 2+d) \mid [A, X] = 0\}$ where A stands for any AdS NKV (see also [10]). We already know that this is true for certain specific choices of A_μ , so we only need to prove that it is true for any other choice. An AdS_{d+3} NKV corresponds to a NKV of the embedding space-time that belongs to $\text{so}(2, d+2)$. Therefore any NKV can be written as the $\text{so}(2, d+2)$ rotation of some preferred one. This preferred Killing vector can be chosen to be the one that commutes with the Schrödinger algebra. Hence they all do. It follows that the metric $\bar{g}_{\mu\nu}$ has at least the Schrödinger algebra as its isometry group. Finally, we know from [16] that a metric admitting the Schrödinger algebra as (possibly part of) its isometry group is either a Schrödinger or an AdS space-time. Since in (5) the metric $g_{\mu\nu}$ is AdS and the shift is not a diffeomorphism we conclude that $\bar{g}_{\mu\nu}$ must be a Schrödinger space-time. \square

We want to take advantage of the Kerr–Schild form of the metric (5) together with the fact that the AdS metric $g_{\mu\nu}$ admits a Fefferman–Graham (FG) expansion to construct an expansion for the Schrödinger space. To that end, we first need to figure out what the most general null Killing vector field on AdS is when expressed in a FG coordinate system. Therefore, we take a pure AdS space in FG coordinates

$$g_{\mu\nu} dx^\mu dx^\nu = \frac{dr^2}{r^2} + \frac{1}{r^2} (g_{(0)ab} + r^2 g_{(2)ab} + r^4 g_{(4)ab}) dx^a dx^b, \quad (7)$$

where³ for $d > 0$ $g_{(2)ab}$ and $g_{(4)ab}$ are fully given in terms of $g_{(0)ab}$, a conformally flat representative of the conformal boundary [17]. We then solve the Killing equations $\nabla_{(\mu} A_{\nu)} = 0$ in this background and

² Note that because $A^2 = 0$ we do not need to specify which metric has been used to lower the index on A^μ .

³ The case $d = 0$ will be presented in [14].

find that the 1-form A_μ is given by

$$A_r = r^{-1} \sigma, \quad (8)$$

$$A_a = g_{ab} A_{(0)}^b - \frac{1}{2} \partial_a \sigma - \frac{r^2}{4} g_{(2)a}{}^b \partial_b \sigma, \quad (9)$$

where $\sigma = (d+2)^{-1} \nabla_a^{(0)} A_{(0)}^a$ and the boundary vector field $A_{(0)}^a$ must satisfy the conformal Killing equation

$$\nabla_a^{(0)} A_{(0)b} + \nabla_b^{(0)} A_{(0)a} = 2\sigma g_{(0)ab}. \quad (10)$$

Here $\nabla^{(0)}$ denotes the covariant derivative with respect to $g_{(0)ab}$ and $A_{(0)a} = g_{(0)ab} A_{(0)}^b$. We note that for $d > 0$ equations (8) and (9) form the most general solution to the Killing equations on a locally AdS space-time and are exact to all orders. However, we are interested only in null Killing vectors. We therefore have to additionally impose the extra condition $A^2 = 0$. To lowest order this translates into the condition $g_{(0)ab} A_{(0)}^a A_{(0)}^b = 0$ which means that one should only consider null conformal Killing vectors $A_{(0)}^a$. Further, since any null Killing vector on AdS is automatically hypersurface orthogonal as can be seen by using the Raychaudhuri equation, it follows, from the leading term in the FG expansion of $A_{[\mu} \nabla_\nu A_{\rho]} = 0$, that $A_{(0)}^a$ is also hypersurface orthogonal with respect to the boundary metric $g_{(0)ab}$. It can be shown [14] that for a conformally flat $g_{(0)ab}$ and a hypersurface orthogonal null conformal Killing vector $A_{(0)}^a$ the higher order terms in the condition $A^2 = 0$ are automatically satisfied and thus do not impose any further restrictions on $A_{(0)}^a$.

4 Locally Schrödinger space-times and the Schrödinger boundary

As already mentioned in the introduction this section was not part of the presentation given at the *XVII European Workshop on String Theory 2011* in Padova. We will now make our way towards a definition of locally Schrödinger space-times by working by analogy with locally AdS spaces. In doing so, we will obtain a definition for the Schrödinger boundary in terms of a defining function. We start by computing the Riemann tensor for a pure Schrödinger space from (5)

$$\bar{R}_{\mu\nu\rho\sigma} = -\bar{g}_{\mu\rho} \bar{g}_{\nu\sigma} + \bar{g}_{\mu\sigma} \bar{g}_{\nu\rho} + \bar{g}_{\mu\rho} A_\nu A_\sigma - \bar{g}_{\nu\rho} A_\mu A_\sigma + \bar{g}_{\nu\sigma} A_\mu A_\rho - \bar{g}_{\mu\sigma} A_\nu A_\rho + \frac{3}{4} F_{\mu\nu} F_{\rho\sigma}, \quad (11)$$

using that A^μ is a hypersurface orthogonal null Killing vector⁴ and denoting $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. We use this result to introduce the notion of a locally Schrödinger space-time, in analogy with locally AdS space-times, as those metrics $\bar{g}_{\mu\nu}$ satisfying (11) as well as admitting (locally) a null Killing vector A^μ . A straightforward property of the Schrödinger Riemann tensor we will make use of is that upon contraction with A^μ it can be seen to behave as an AdS Riemann tensor

$$\bar{R}_{\mu\nu\rho\sigma} A^\sigma = (-\bar{g}_{\mu\rho} \bar{g}_{\nu\sigma} + \bar{g}_{\mu\sigma} \bar{g}_{\nu\rho}) A^\sigma. \quad (12)$$

With the use of (5) and (12) we define a Schrödinger boundary in terms of a defining function $\Omega(r, x)$. First, let $\Omega = 0$ denote the location of the Schrödinger boundary. We will consider a conformal rescaling of both the AdS and the Schrödinger metric. Denoting by a tilde the conformally rescaled metrics we have $\tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}$ and $\tilde{\bar{g}}_{\mu\nu} = \Omega^2 \bar{g}_{\mu\nu}$. Under such a conformal rescaling equation (5) transforms as

$$\tilde{\bar{g}}_{\mu\nu} = \tilde{g}_{\mu\nu} - \Omega^{-2} \tilde{A}_\mu \tilde{A}_\nu, \quad (13)$$

where $\tilde{A}_\mu = \tilde{g}_{\mu\nu} A^\nu$. Following the same procedure as in [18] we rewrite the Schrödinger Riemann tensor in terms of the conformally rescaled quantities. To leading order in a near boundary expansion we find

$$\bar{R}_{\mu\nu\rho\sigma} = \Omega^{-4} (-\tilde{\bar{g}}_{\mu\rho} \tilde{\bar{g}}_{\nu\sigma} + \tilde{\bar{g}}_{\mu\sigma} \tilde{\bar{g}}_{\nu\rho}) \tilde{\bar{g}}^{\kappa\tau} \partial_\kappa \Omega \partial_\tau \Omega + \mathcal{O}(\Omega^{-3}). \quad (14)$$

⁴ Note that A^μ is actually a hypersurface orthogonal null Killing vector of $g_{\mu\nu}$ as well as of $\bar{g}_{\mu\nu}$.

It then directly follows from (12) and the contraction of (14) with A^σ that

$$\Omega^{-2} \bar{g}^{\kappa\tau} \partial_\kappa \Omega \partial_\tau \Omega \Big|_{\Omega=0} = 1. \quad (15)$$

On the other hand, since $\Omega^{-2} \tilde{g}_{\mu\nu}$ is an AdS metric, Ω must also satisfy the properties of an AdS defining function implying for similar reasons as above that $\Omega^{-2} g^{\kappa\tau} \partial_\kappa \Omega \partial_\tau \Omega \Big|_{\Omega=0} = 1$. Hence, by substituting the expression for the inverse Schrödinger metric $\bar{g}^{\mu\nu} = g^{\mu\nu} + A^\mu A^\nu$ into (15) we learn that

$$\Omega^{-2} (A^\mu \partial_\mu \Omega)^2 \Big|_{\Omega=0} = 0. \quad (16)$$

As $\partial_\mu \Omega$ is normal to the boundary we arrive at the important conclusion that A^μ must necessarily be tangential to the Schrödinger boundary⁵.

Let us now go back to the AdS space point of view. The conformal rescaling $\tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}$ transforms the Killing equation for the null Killing vector A^μ into

$$\tilde{\nabla}_\mu \tilde{A}_\nu + \tilde{\nabla}_\nu \tilde{A}_\mu = 2 \tilde{g}_{\mu\nu} A^\rho \partial_\rho \log \Omega, \quad (17)$$

where $\tilde{A}_\mu = \tilde{g}_{\mu\rho} A^\rho$. The projection of (17) onto the boundary is

$$\tilde{h}_\mu{}^\rho \tilde{h}_\nu{}^\sigma \tilde{\nabla}_\rho (\tilde{h}_\sigma{}^\tau \tilde{A}_\tau) + \tilde{h}_\nu{}^\rho \tilde{h}_\mu{}^\sigma \tilde{\nabla}_\rho (\tilde{h}_\sigma{}^\tau \tilde{A}_\tau) = 2 \tilde{h}_{\mu\nu} A^\rho \partial_\rho \log \Omega - 2 A^\tau \tilde{n}_\tau \tilde{K}_{\mu\nu}, \quad (18)$$

where \tilde{n}^μ denotes the unit normal to the AdS boundary with respect to the rescaled metric $\tilde{g}_{\mu\nu}$, the boundary projector $\tilde{h}_\rho{}^\mu$ is given by $\tilde{h}_\rho{}^\mu = \delta_\rho^\mu - \tilde{n}_\rho \tilde{n}^\mu$ and $\tilde{K}_{\mu\nu} = \tilde{h}_\mu{}^\sigma \tilde{\nabla}_\sigma \tilde{n}_\nu$ is the extrinsic curvature. Evaluating (18) at $\Omega = 0$ using (16) and the fact that $\tilde{n}_\mu = \partial_\mu \Omega$ near $\Omega = 0$ we conclude that the boundary projected vector $\tilde{h}_\mu{}^\tau \tilde{A}_\tau$ is an AdS boundary Killing vector. It is straightforward to show that it is also null with respect to the AdS boundary metric.

As a result we observe that the condition (16) restricts the AdS boundary metrics to those admitting a null Killing vector. This condition restricts the expansion given by (8) and (9). In other words we find that the AdS decomposition of the Schrödinger metric (5) relates the Schrödinger boundary to those AdS boundaries whose metric admits a null Killing vector. In the previous section starting from (5) we could have used any AdS Fefferman–Graham coordinate system to perform the shift to Schrödinger. This is certainly possible but in this way we would also consider coordinates such as global AdS coordinates and write the Schrödinger metric in terms of those. Without any further restrictions this approach gives expressions for the Schrödinger metric that appear to be in Fefferman–Graham form whereas they are really not unless we had imposed from the start to only take into account those AdS coordinates satisfying (16). For example had we chosen global AdS coordinates we would end up with an expression for the Schrödinger metric in which the Schrödinger boundary is no longer manifest because A^μ is not tangent to the global AdS boundary.

We finally proceed to the construction of a Fefferman–Graham expansion for locally Schrödinger space-times based on these new observations. Using (5), the well-known Fefferman–Graham expansions for locally AdS space-times [17] and the results of the previous section together with the new restrictions on the AdS boundary metric to admit a null Killing vector we obtain for $d > 0$

$$d\bar{s}^2 = \frac{dr^2}{r^2} + \left(\frac{\tilde{h}_{ab}}{r^2} - \frac{\tilde{A}_a \tilde{A}_b}{r^4} \right) dx^a dx^b, \quad (19)$$

$$\tilde{h}_{ab} = g_{(0)ab} + r^2 g_{(2)ab} + \frac{r^4}{4} g_{(2)a}{}^c g_{(2)c}{}^b \quad (20)$$

$$A_a = r^{-2} \tilde{A}_a = r^{-2} \tilde{h}_{ab} A_{(0)}^b, \quad (21)$$

$$A_r = 0, \quad (22)$$

⁵ We mention that the so-called lightlike lines on the Schrödinger space are geodesics whose tangent is precisely A^μ . In [6] we showed that these lightlike lines play a key role in describing the non-relativistic causal structure of the Schrödinger space-time. The fact they are tangential to the boundary is strongly indicative of the boundary inheriting this non-relativistic causal structure.

in which $g_{(0)ab}$ is a conformally flat representative of the conformal boundary that admits $A_{(0)}^a$ as a null Killing vector and $g_{(2)ab}$ is given by

$$g_{(2)ab} = -\frac{1}{d} \left(R_{(0)ab} - \frac{R_{(0)}}{2(d+1)} g_{(0)ab} \right). \quad (23)$$

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